MT Classes Math Olympiad

Ages 11–12 • 90 minutes • October 2026 • English

Multiple-choice problems

Each problem has several answer choices. Choose the one correct answer and write it on the answer sheet.

Problem 1 (3 points). Misha writes to seven friends every day and to each of nine other friends every other day. Today he wrote to ten friends. How many friends will he write to tomorrow?

(a) 6 **(b)** 7 **(c)** 9 **(d)** 13 **(e)** 16

Answer: 13

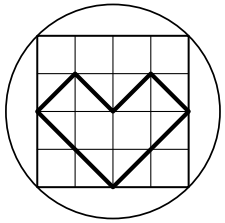
Solution. Besides the seven friends Misha writes to every day, today he wrote to three of the nine friends he writes to every other day. So tomorrow he will write to the remaining six of those nine and to the seven daily correspondents. In total

$$6 + 7 = 13.$$

□

Problem 2 (3 points).

A square divided into 16 equal squares is inscribed in a circle of diameter 10 cm. What is the perimeter of the figure drawn with the thick line?



(a) 8 **(b)** 16 **(c)** 20 **(d)** 24 **(e)** 30

Answer: 20

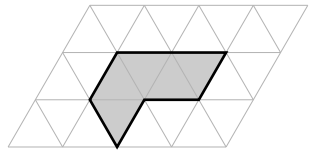
Solution. The diagonal of the large square is a diameter of the circle and equals 10 cm. It consists of four diagonals of the small cells. The thick line consists of eight such diagonals, so its length is twice as large and equals 20 cm. □

Short-answer problems

Write only the final answer (a number, an expression, an example, a short statement — as required in the problem). No solution is needed: only the answer is checked.

Problem 3 (5 points).

On a triangular grid, figures are made of small equilateral triangles with side 1. The figure on the right consists of five such triangles, has perimeter 7 and six vertices. The vertices of a figure are the points where its boundary changes direction.



Shade 10 triangles joined to each other along their sides so that the result is a figure without holes, with perimeter 12 and nine vertices.

Answer: See the figure in the solution.

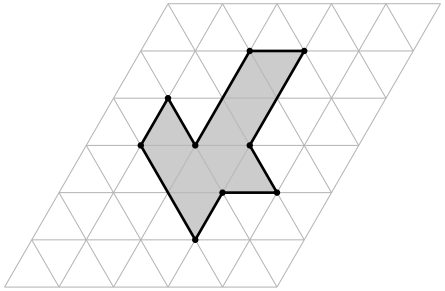
Solution. If two shaded triangles share a side, this side is not part of the figure's perimeter. So each common side reduces the sum of the perimeters of the separate triangles by 2.

Ten separate triangles have a total perimeter of 30. For the perimeter of the figure to be 12, there must be

$$\frac{30 - 12}{2} = 9$$

common sides. This is possible, for example, if each new triangle is attached to the previous ones along exactly one side.

One suitable example is shown in the figure. The nine vertices of the figure are marked with dots.



The figure consists of ten triangles and has nine common sides, so its perimeter is $30 - 2 \cdot 9 = 12$. The direction of the boundary changes at the nine marked points. □

Problem 4 (5 points). Lydia writes several single-digit numbers on the board, then adds them from left to right and writes down each intermediate sum. For example, starting with the numbers 3, 9, 5, she ends up with 3, 9, 5, 12, 17 on the board, in which the digits 1, 2, 3, 5, 7, 9 appear.

Give an example of four starting numbers such that after the computations every digit from 1 to 9 appears on the board.

Answer: For example, 4, 5, 8, 6.

Solution. It is convenient to first choose the intermediate sums and then recover the initial numbers. If the difference of two consecutive sums is a single-digit number, then exactly this number can be put next in the sequence. The first sum must be split into two single-digit addends.

Choose, for example, the intermediate sums 17 and 23. The digits 1, 2, 3 and 7 appear in them, and their difference is $23 - 17 = 6$. Before 17 it is convenient to obtain the sum 9: then the next initial number will be $17 - 9 = 8$. Finally, $9 = 4 + 5$.

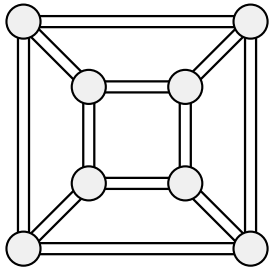
This way we get the sequence

$$4,\ 5,\ 8,\ 6.$$

Other suitable examples can be built in the same way, choosing other intermediate sums and recovering the initial numbers from their differences. □

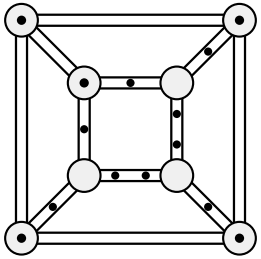
Problem 5 (5 points).

A park has 8 circular plazas connected by paths, as shown in the figure. The park director wants to install 14 lamps so that every path is lit by the same number of lamps.



At most one lamp can be placed on each circular plaza, and any number of lamps along the paths. A lamp placed on a plaza lights all the paths leaving that plaza. A lamp placed on a path lights only that path. Give an example of an arrangement of the lamps.

Answer: For example, the lamps can be arranged as shown in the figure. The lamps are marked with black dots.



Solution. We aim for every path to be lit by exactly two lamps. Put one lamp on each of the five plazas marked in the figure. Then on each path put as many lamps as needed so that together with the lamps at its ends there are two.

On the four outer paths no additional lamps are needed. On the four inner paths we need, respectively, 1, 2, 2 and 1 lamps, and on the paths joining the outer and inner squares, respectively 0, 1, 1 and 1 lamps. In total, the number of lamps placed along the paths will be

$$1 + 2 + 2 + 1 + 0 + 1 + 1 + 1 = 9$$

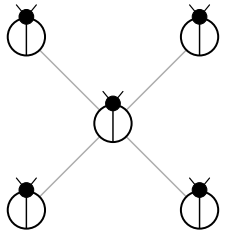
lamps, and together with the five lamps on the plazas we get 14 lamps. Every path is lit by exactly two lamps. □

Full-solution problems

Write a complete solution: your reasoning, justifications and proof (where required). Both the answer and the reasoning are assessed.

Problem 6 (7 points).

Five ladybugs arranged themselves as shown in the figure. It turned out that the three ladybugs on one diagonal have 15 black dots in total, and the three on the other diagonal have 16 dots in total. The two ladybugs on the left have 14 dots together, and all five together have 26 dots. How many dots do the two ladybugs on the right have together?



Answer: 7

Solution. Add the numbers of dots on the two diagonals. We get

$$15 + 16 = 31.$$

Here the dots of the four corner ladybugs are counted once each, and the dots of the central one, twice. So, compared with the total number of dots, we counted the dots of the central ladybug one extra time. Hence it has

$$31 - 26 = 5$$

dots.

The two ladybugs on the left have 14 dots together. So the two left ladybugs and the central one have in total

$$14 + 5 = 19$$

dots. All five ladybugs have 26 dots, so the two ladybugs on the right have

$$26 - 19 = 7$$

dots. □

Problem 7 (7 points). In a Transfiguration lesson, Professor McGonagall waved her wand, and a row of distinct two-digit numbers appeared on the board by itself. The students were given a task: Harry was to erase from the board all numbers divisible by 2, Ron all numbers divisible by 3, Hermione all numbers divisible by 5, and Neville all numbers divisible by 7.

The four of them did the task one after another, but it is unknown in what order. In the end, Harry erased the numbers 32 and 56, Ron erased 24, 33 and 45, and Hermione erased 20, 25 and 35.

It is known that Neville erased exactly three numbers. Which ones?

Answer: 49, 77 and 91

- 45 is divisible by 3 and by 5, but it was erased by Ron. So Ron went before Hermione;

- 20 is divisible by 2 and by 5, but it was erased by Hermione. So Hermione went before Harry;
- 56 is divisible by 2 and by 7, but it was erased by Harry. So Harry went before Neville.

Solution. We get the only possible order:

$$\text{Ron} \to \text{Hermione} \to \text{Harry} \to \text{Neville}.$$

Therefore Neville could erase only numbers that are divisible by 7 but not divisible by 2, 3 or 5: otherwise one of the other students would have erased them earlier.

Among the two-digit multiples of 7:

$$14,\ 21,\ 28,\ 35,\ 42,\ 49,\ 56,\ 63,\ 70,\ 77,\ 84,\ 91,\ 98.$$

Of these, only 49, 77 and 91 are divisible by none of 2, 3, 5. There are exactly three such numbers, so these are the ones Neville erased. □